Number Sense and Reasoning (3 - 8)

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99
+ 3

112

Number Sense

- Suggests patterns or relationships between the numbers
- Supports a solution path
- Determines reasonableness of solutions

Number Sense and Struggle

- Suggests patterns or relationships between the numbers
- Supports a solution path
- Determines reasonableness of solutions
- Underdeveloped number sense contributes to struggle (numbers overwhelm, etc)

Number Sense and MTSS

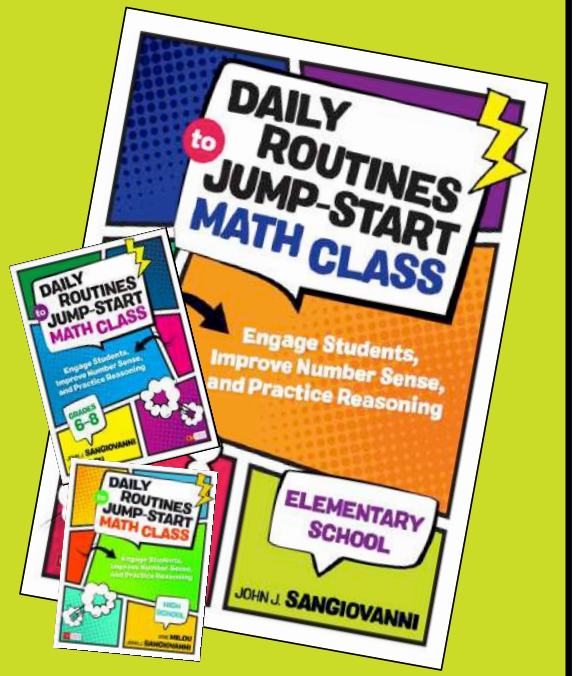
- Developing number sense is often overlooked in systems of support and intervention.

- Opportunities for developing number sense benefit each and every student

About the Session

- Identify the value of daily number sense and reasoning
- Start a collection daily routines

www.tinyurl.com/howardcountymath



Daily Routines to Jump-Start Math Class **K-5** (Corwin, 2019)
Daily Routines to Jump-Start Math Class **6-8** (Corwin, 2018)
Daily Routines to Jump-Start Math Class **9-12** (Corwin, 2018)

How many pieces of gum are in the picture?

How many are on the bookcase?

If this is 15 books…

A full stadium holds 65,000….

How many sit here?

How many people can ride the Ferris Wheel at one time?

How many carts are on the Ferris Wheel?

Are there more or less than 100?

What is a number that's too big?

What's a number that's too small?

Are there more or less than 25 brownies?

A brownie is 100 calories. How many calories are on the plate?

About how many cookies are there?

How many legs?

A number that's **too big**…

A number that's **too small**…

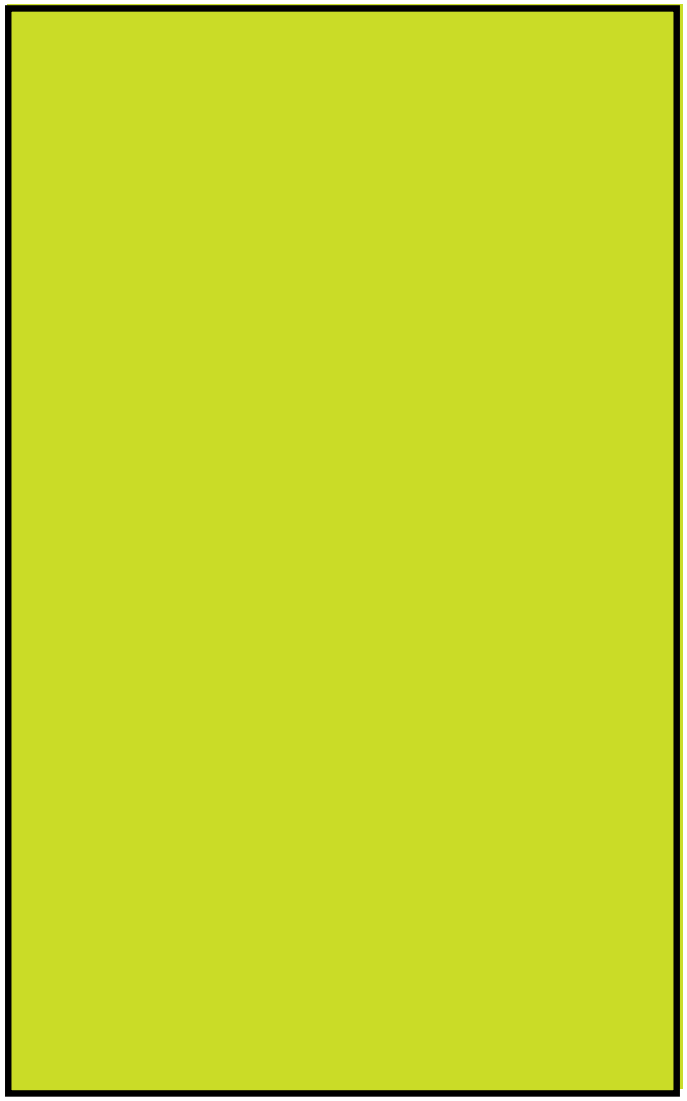
If I wanted a sprinkled donut which box should I choose?

What is a question you might ask?

Picture Perfect

- Pose a picture.
- Have students notice mathematics or reason and estimate to respond to a prompt.

What is number sense?



Describe a student who has number sense.

What makes this student come to mind?

What is Number Sense?

- an awareness and understanding about what numbers are
- their relationships,
- their magnitude,
- the relative effect of operating on numbers,
- including the use of mental mathematics and estimation

Fennell and Landis (1994)

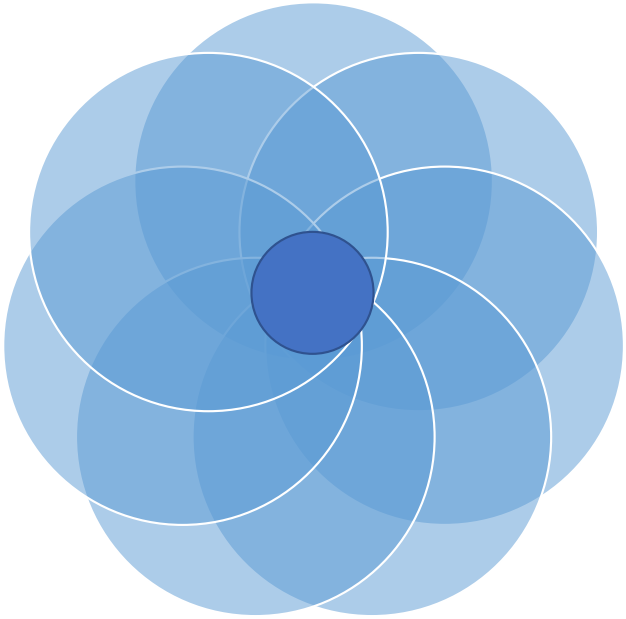
Number Sense

- mental calculation (Hope & Sherrill, 1987; Trafton, 1992);
- computational estimation (for example; Bobis, 1991; Case & Sowder, 1990);
- judging the relative magnitude of numbers (Sowder, 1988);
- recognizing part-whole relationships and place value concepts (Fischer, 1990; Ross, 1989)
- problem solving (Cobb et.al., 1991)

Magnitude

Awareness

Estimation



Flexibility

Mental Math

Reasoning

Decomposition

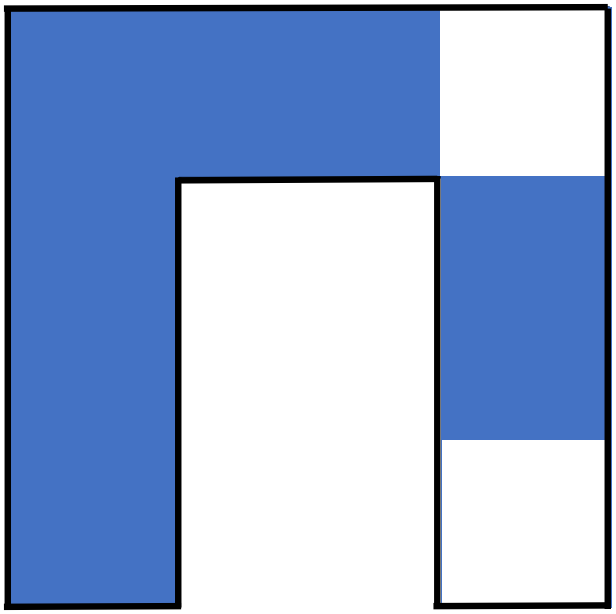
Number Sense

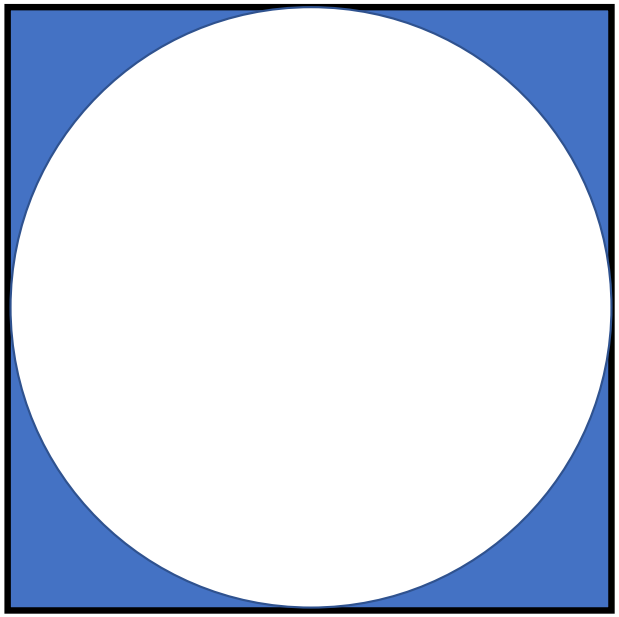
An intuitive understanding of numbers, their magnitude, relationships, and how they are affected by operations.

-Learn NC, University of North Carolina

F. It's About

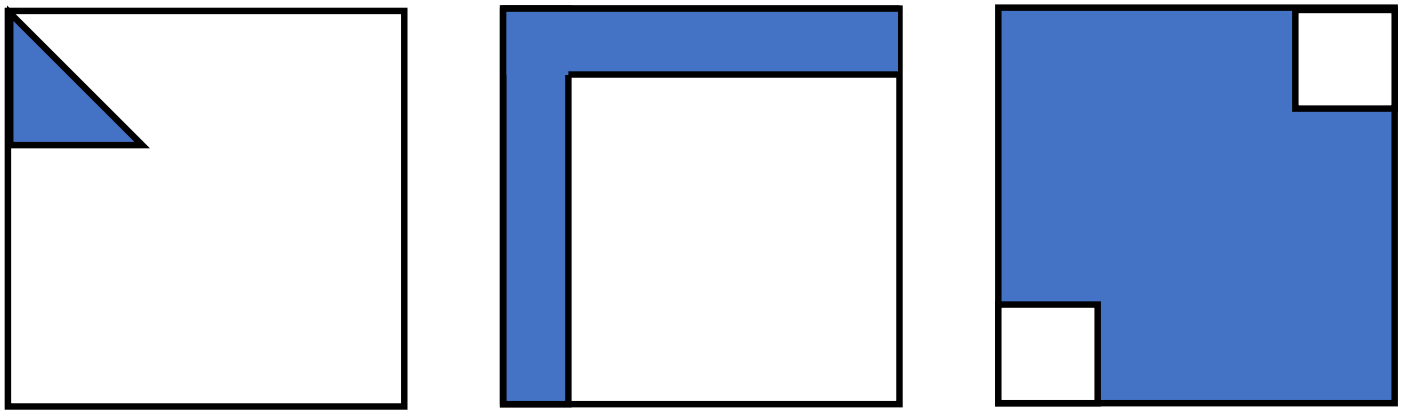
- Estimate the shaded amount of each figure.





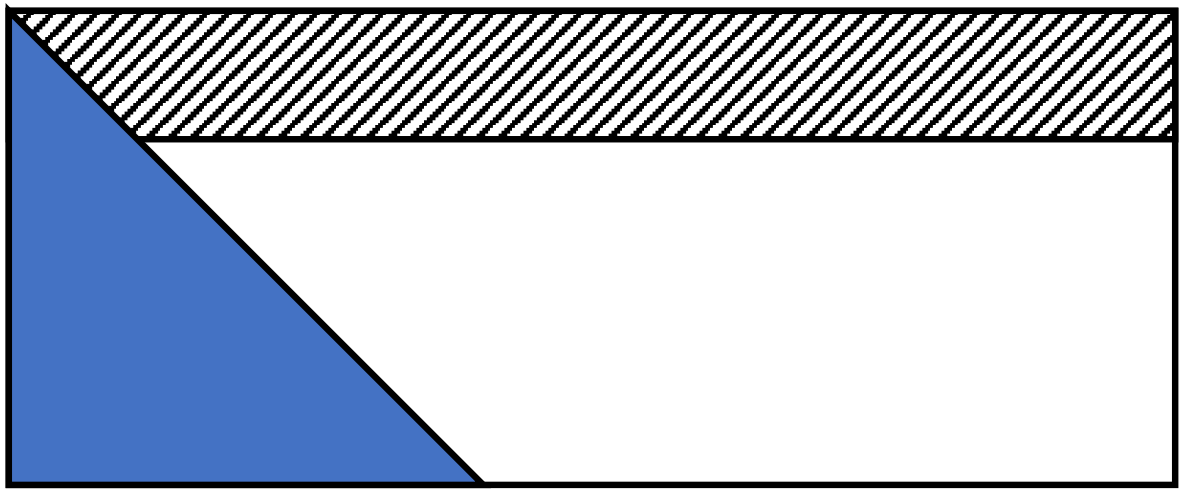
It’s About

- Estimate the shaded amount of each figure.



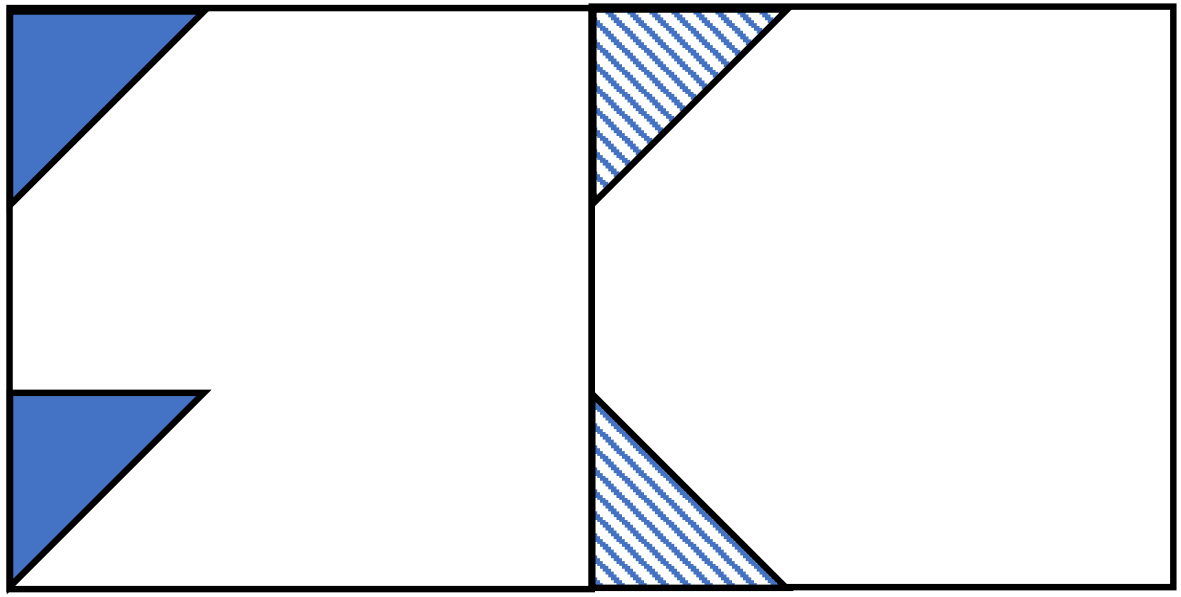
A. It's About

- Estimate the shaded amount of the figure.



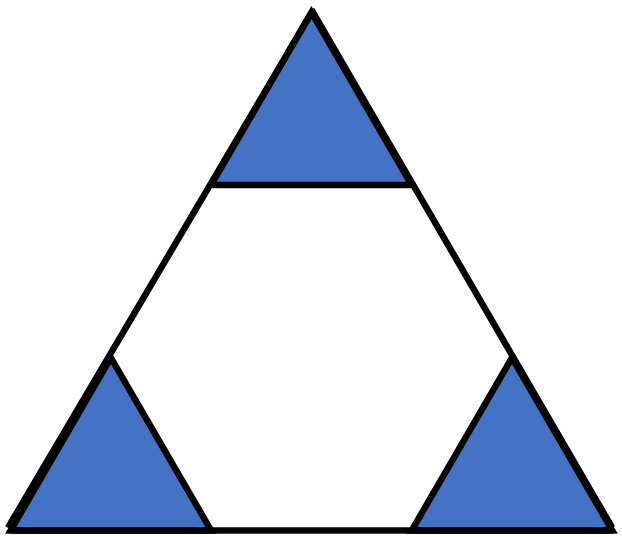
B. It's About

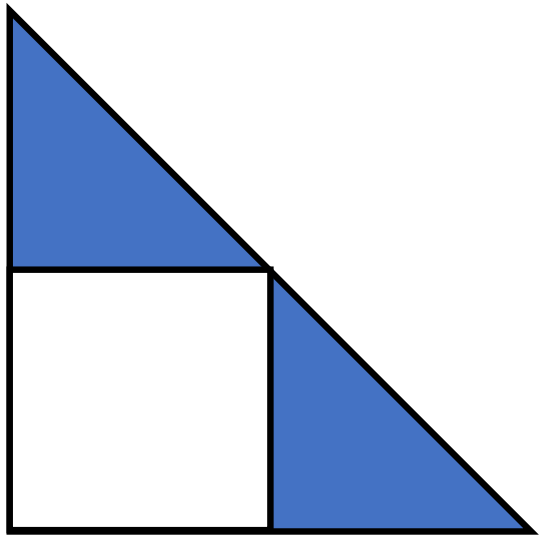
- Estimate the shaded amount of the figure.



C. It's About

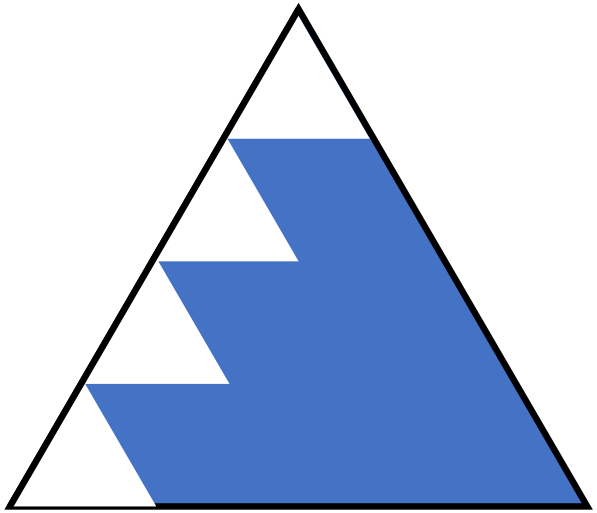
- Estimate the shaded amount of each figure.

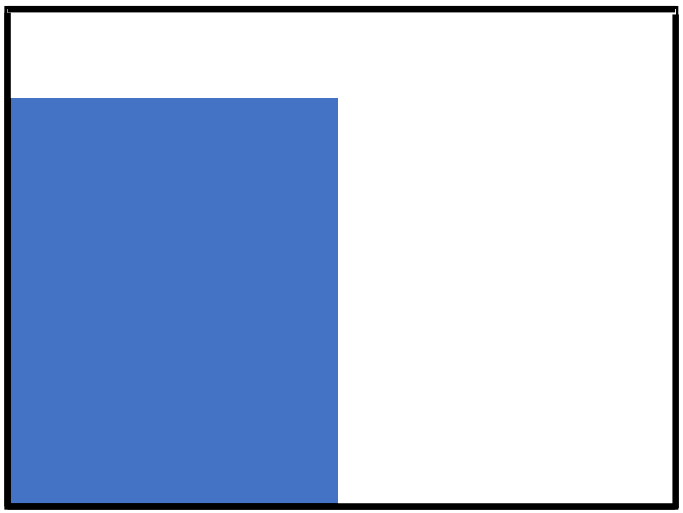




E. It's About

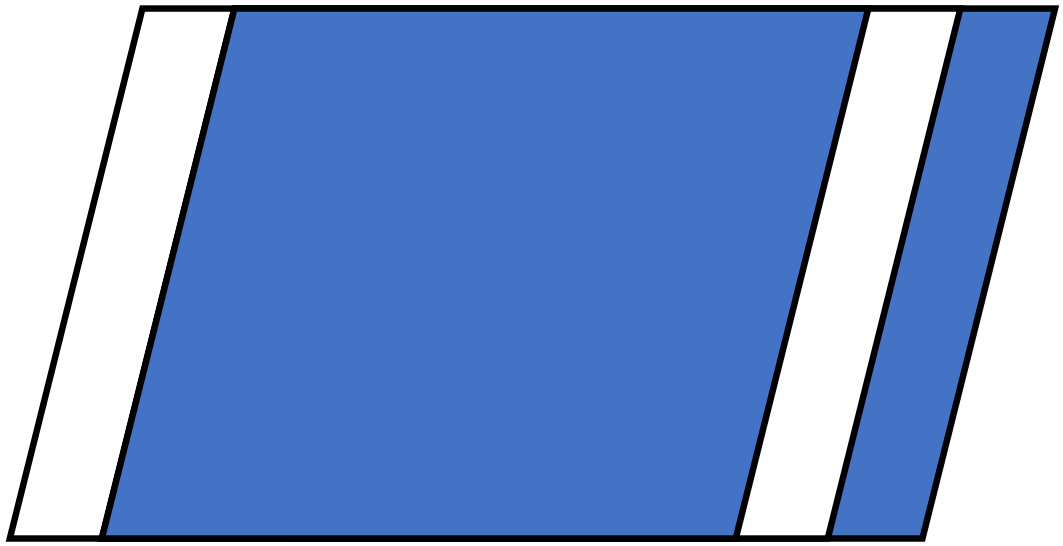
- Estimate the shaded amount of each figure.





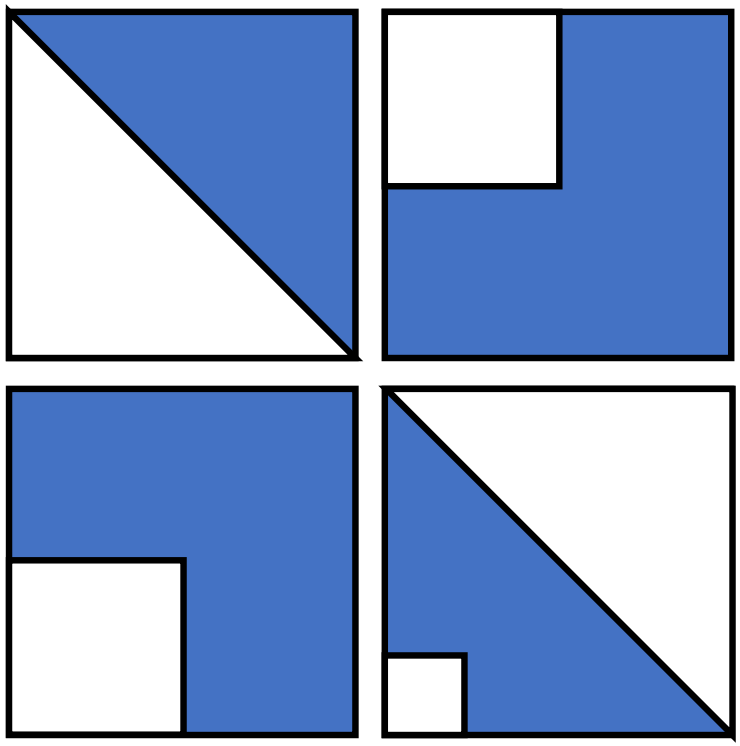
G. It's About

- Estimate the shaded amount of each figure.



H. It's About

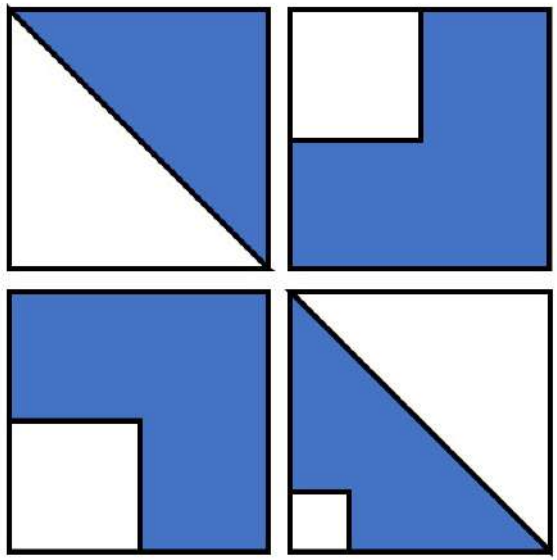
- Estimate the shaded amount of each figure.



It's About

- Provide a shaded region without exact partitions.
- Have students determine the fractional amount shaded.

• Estimate the shaded amount of each figure.



Number Sense is the perfect problem.

It’s not measured.

It’s not a specific standard.

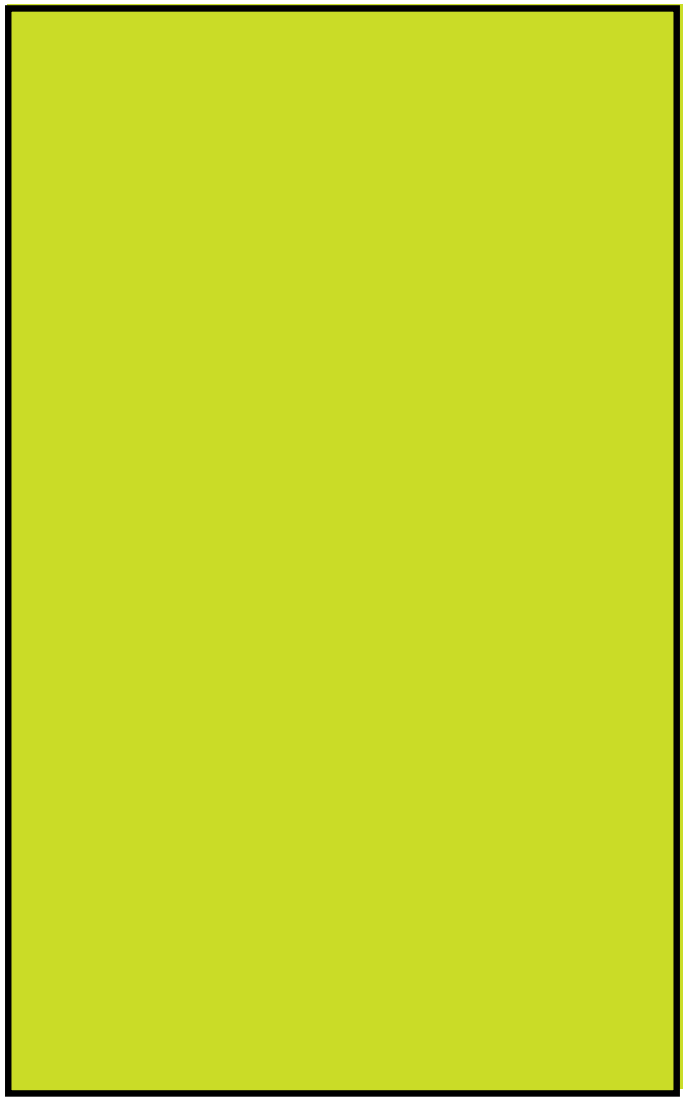
It’s not a lesson.

Number Sense is the perfect problem.

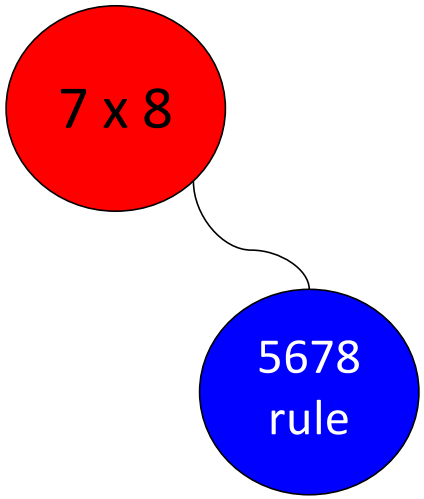
"They just don't have any number sense."

"So, what are you doing about it?"

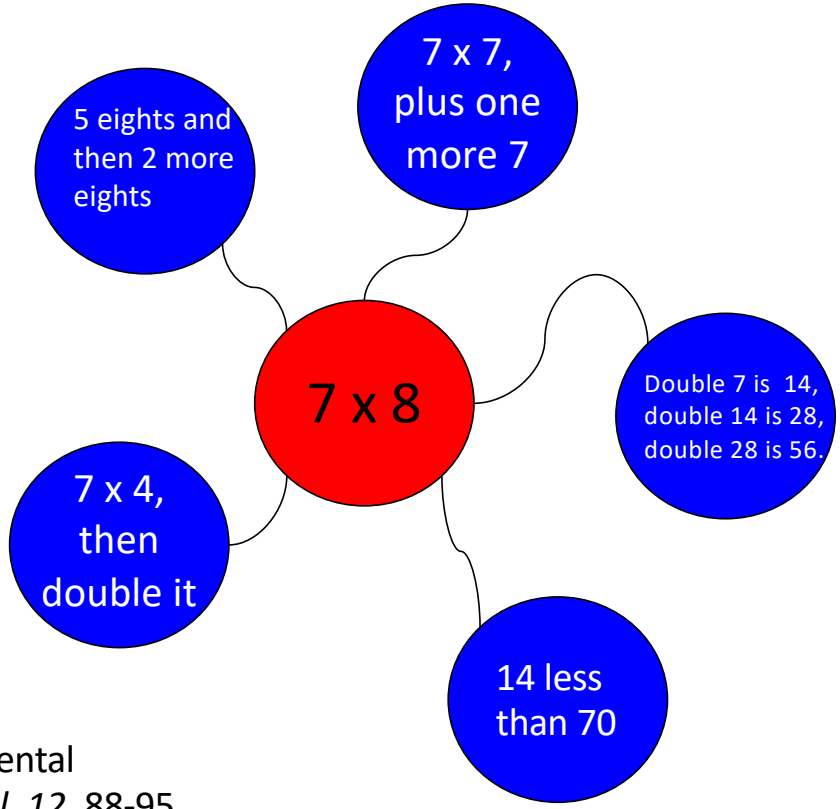
How do we develop understanding and utility?



Instrumental Understanding



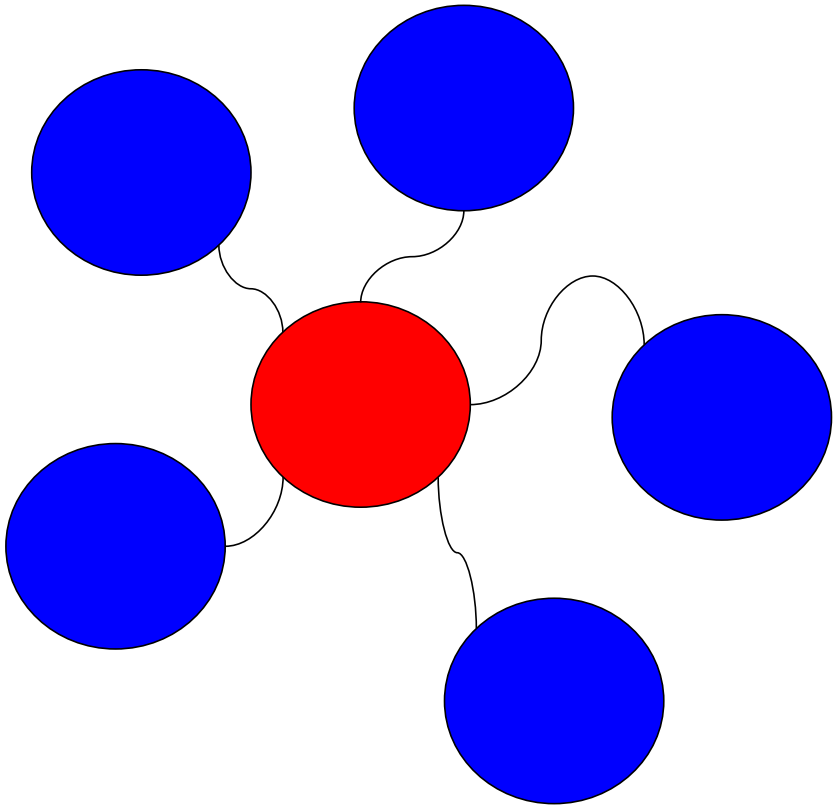
Relational Understanding



Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School, 12*, 88-95.

Developing Relational Understanding

- It is intrinsically rewarding.
- It enhances memory.
- There is less to remember.
- It helps with learning new concepts and procedures.
- It improves problem-solving abilities.
- It is self-generative.
- It improves attitudes and beliefs.



Write a three-digit number.
Match the CONDITION to earn a point.

Give yourself a point if your number

Write a three-digit number.
Match the CONDITION to earn a point.

Give yourself a point if your number

- has 5 in it

Write a three-digit number.
Match the CONDITION to earn a point.

Give yourself a point if your number

- has 5 in it
- Is between 200 and 500

Write a three-digit number.
Match the CONDITION to earn a point.

Give yourself a point if your number

- has 5 in it
- Is between 200 and 500
- Rounds to 700

Write a three-digit number.
Match the CONDITION to earn a point.

Give yourself a point if your number

- has 5 in it
- Is between 200 and 500
- Rounds to 700
- has a sum of digits that are even

Write a three-digit number.
Match the CONDITION to earn a point.

Give yourself a point if your number

- has 5 in it
- Is between 200 and 500
- Rounds to 700
- has a sum of digits that are even
- Is the smallest three-digit number in the class

F.

Write a two-digit decimal number.
Match the **CONDITION** to earn a point.

Your decimal

- Is between 5.0 and 6.0
- Has a two in it
- Has 8 tens
- Has no ones
- Is even

E.

Write a fraction.

Match the CONDITION to earn a point.

Your fraction

- Is more than $\frac{1}{2}$
- Is equal to $\frac{1}{2}$
- Is equivalent to $\frac{2}{3}$
- Is between 1 and 2
- Can be added to $\frac{3}{4}$ to make one whole.

Write a three-digit **INTEGER**.
Match the **CONDITION** to earn a point.

Your number

- has 5 in it
- Is between -200 and -500
- Rounds to 700
- has a sum of digits is a multiple of 4
- Is the smallest three-digit number in the class

Write an expression.
Match the CONDITION to earn a point.

When x is…

- 5 your expression is greater than…
- 1.7 your expression is…
- ¾ your expression is a whole number…
- -8 your expression is positive

Match the CONDITION to earn a point.

Then pass your number to someone else.

Your number

- Rounds to 80.
- Has a 4 in it.
- Is even.
- Is the largest number in the class.
- Is close to 0.

Stand up if you meet the condition.

Sit down if you don’t meet the condition.

Your number

- Has eight ones
- Is larger than 50
- Is between 70 and 90
- Is 20 or more less than your partner.

Condition

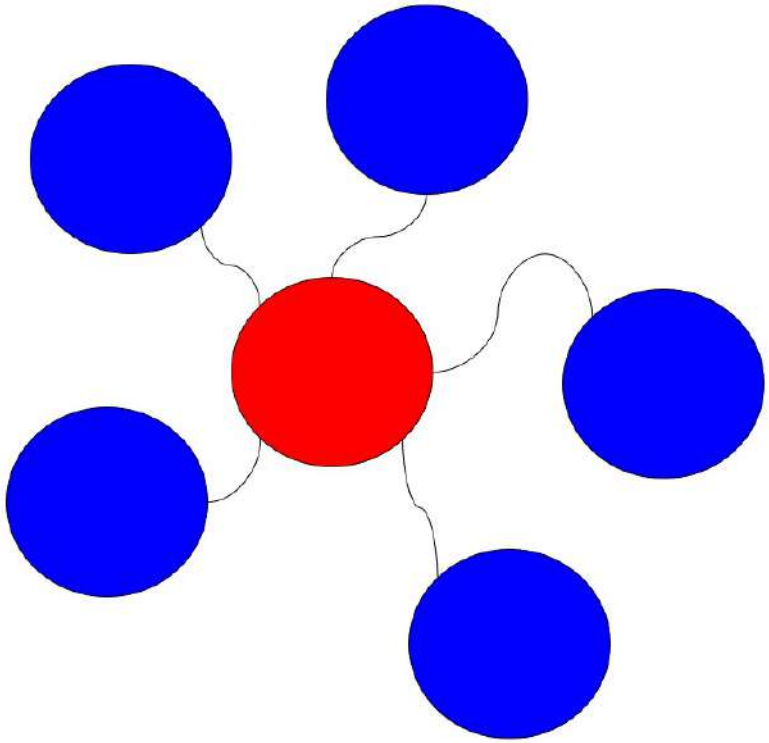
- Students create a number.
- Students earn a point if their number meets each new condition.

Match the CONDITION to earn a point.

Your fraction

- Is more than $\frac{1}{2}$
- Is equal to $\frac{1}{2}$
- Is equivalent to $\frac{2}{3}$
- Is between 1 and 2
- Can be added to $\frac{3}{4}$ to make one whole.

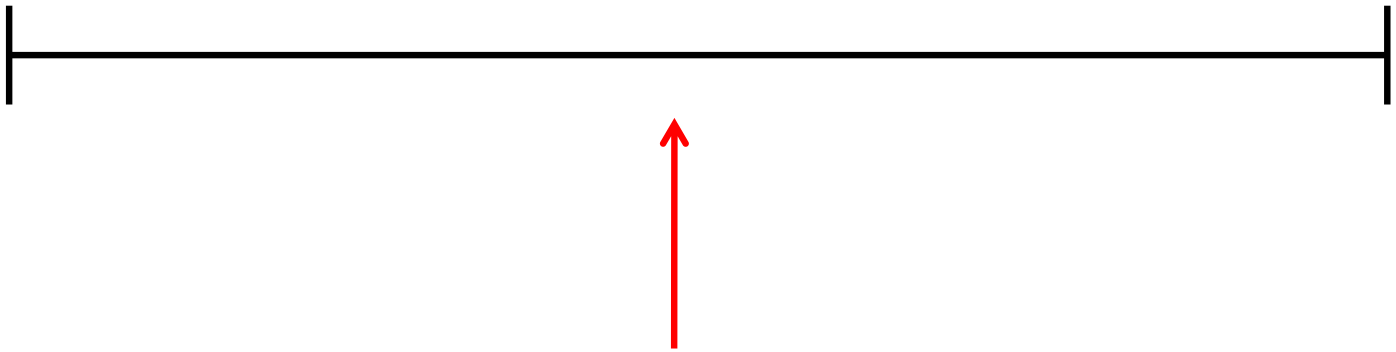
Numbers are related and connected in a variety of ways.



Is This the End?

The arrow is pointing at 3.45.

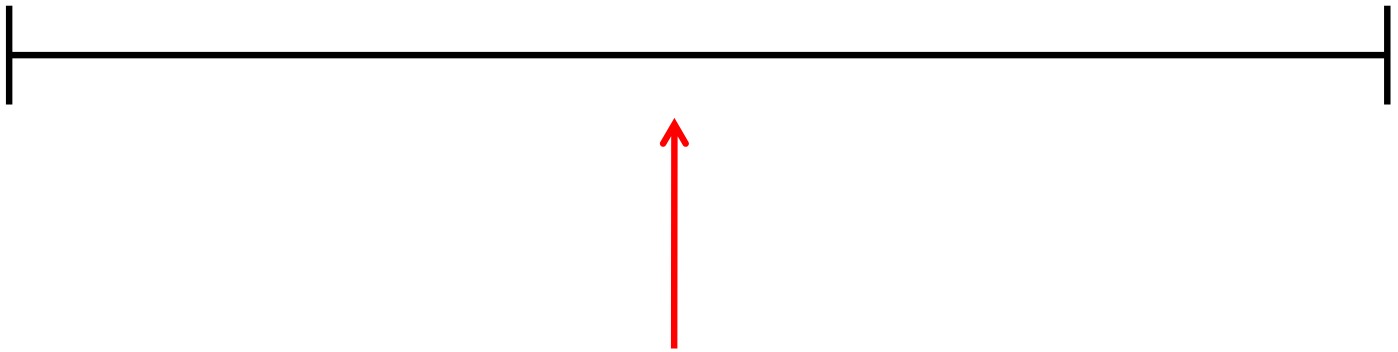
What are the endpoints?



Is This the End?

The arrow is pointing at **3,450**.

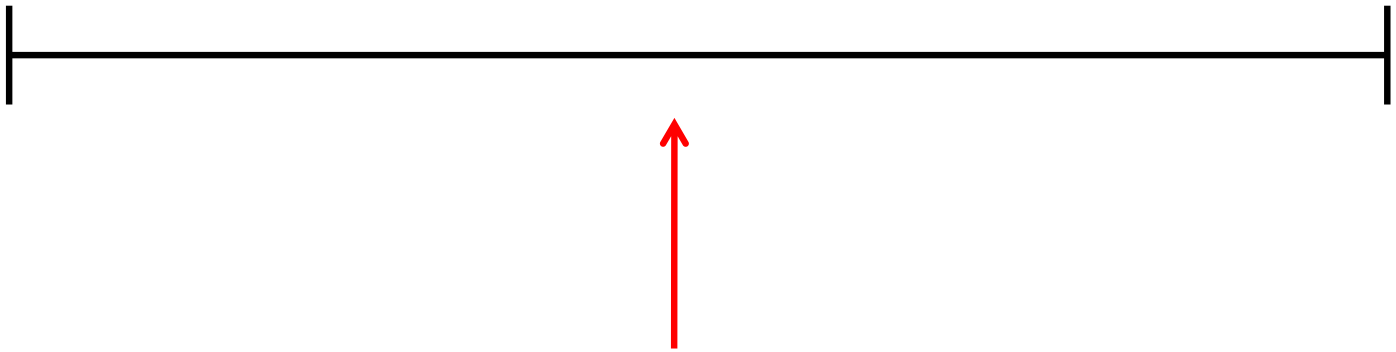
What are the endpoints?



Is This the End?

The arrow is pointing at **34,500**.

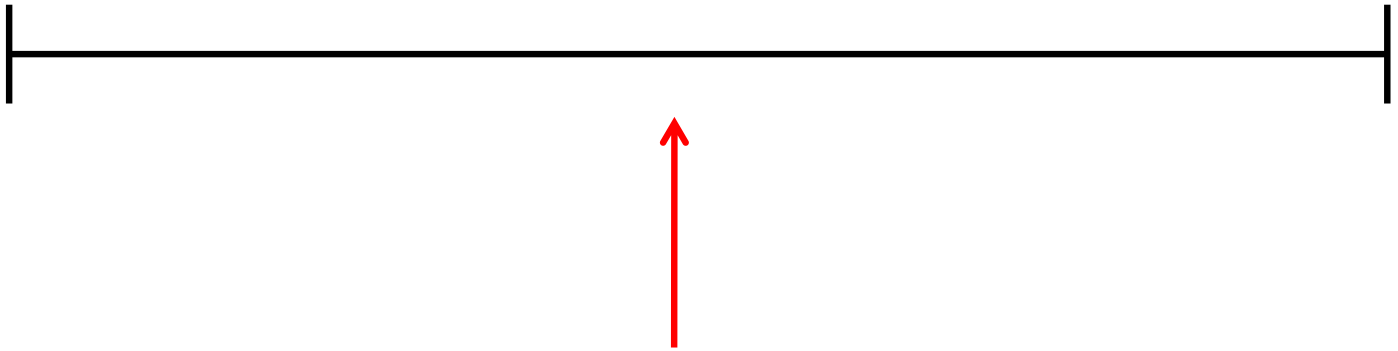
What are the endpoints?



Is This the End?

The arrow is pointing at $\mathbf{2\frac{6}{8}}$.

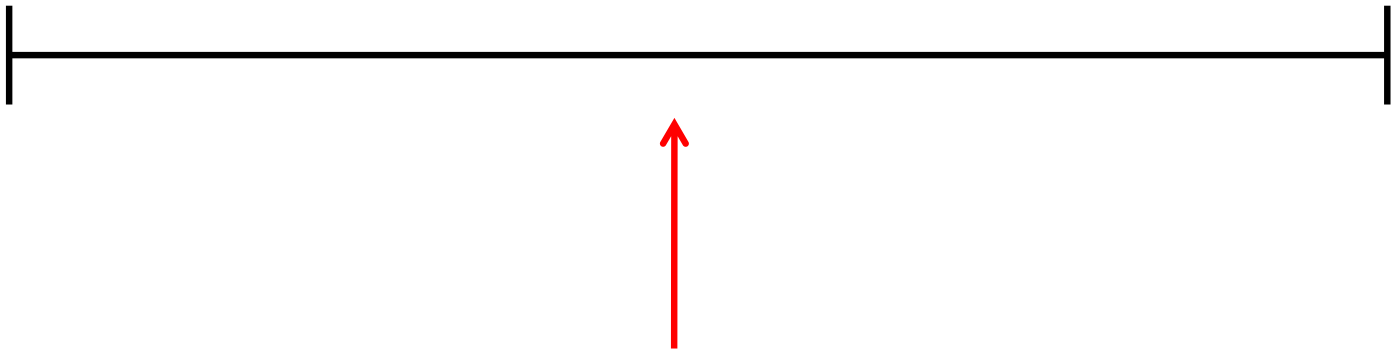
What are the endpoints?



A. Is This the End?

The arrow is pointing at -13.

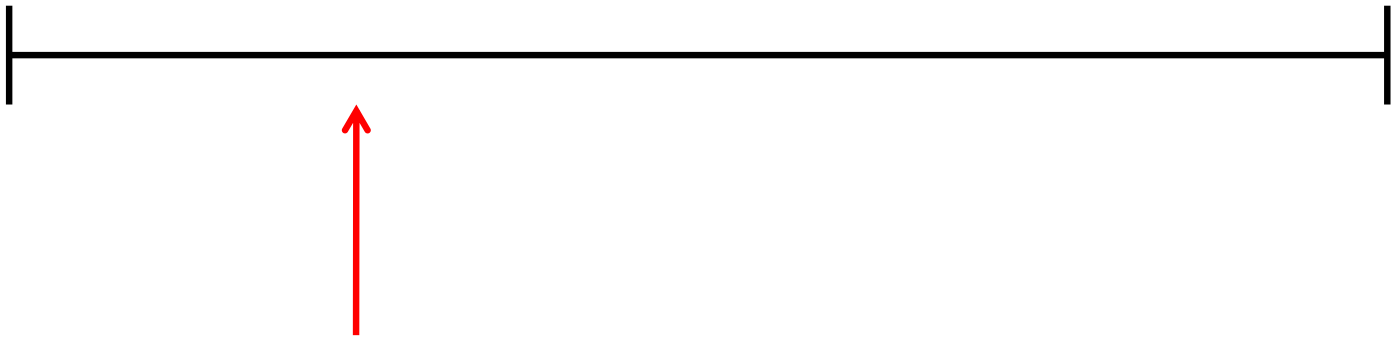
What are the endpoints?



C. Is This the End?

The arrow is pointing at -28.6.

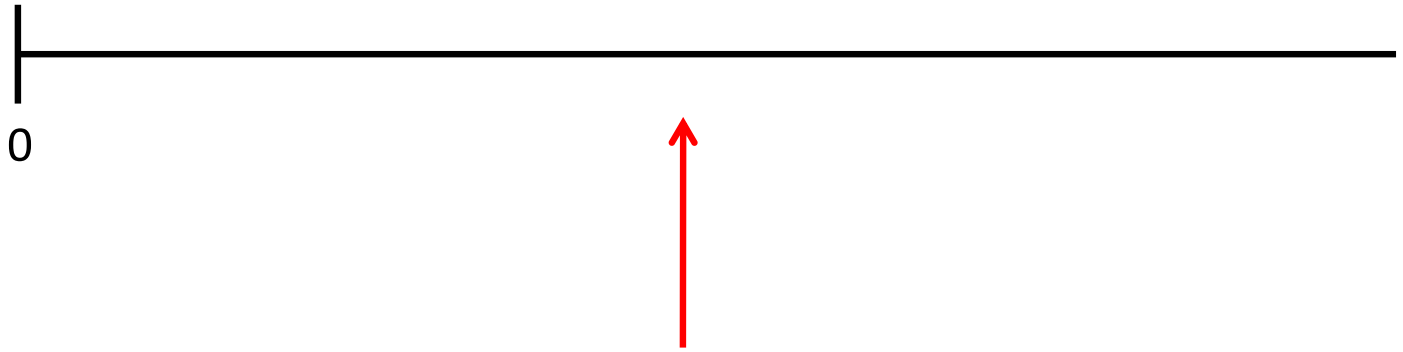
What are the endpoints?



E.

The arrow is pointing at 20.

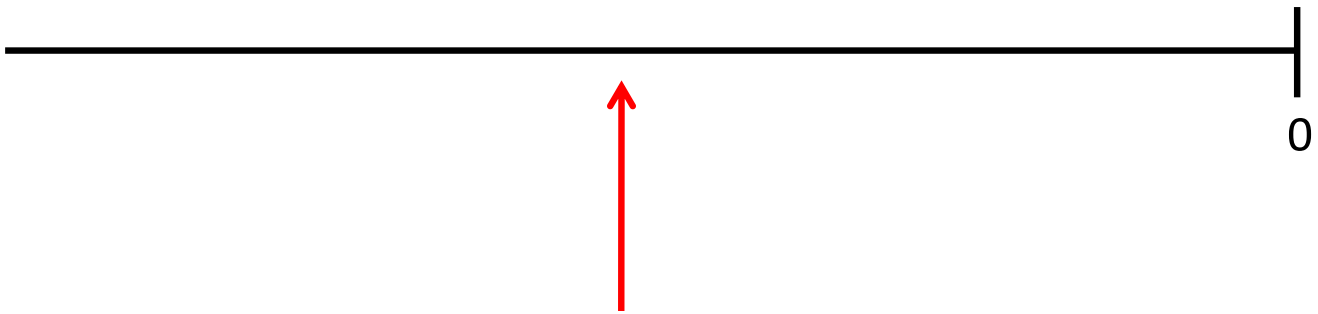
About where is 10? 22? 45?



G. Is This the End?

The arrow is pointing at $-\frac{3}{4}$

About where is -1? $-\frac{1}{2}$? -0.25?

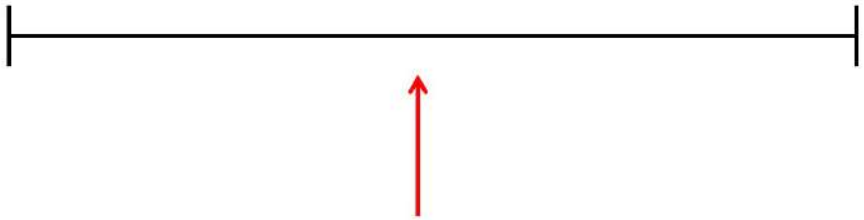


Where's the End?

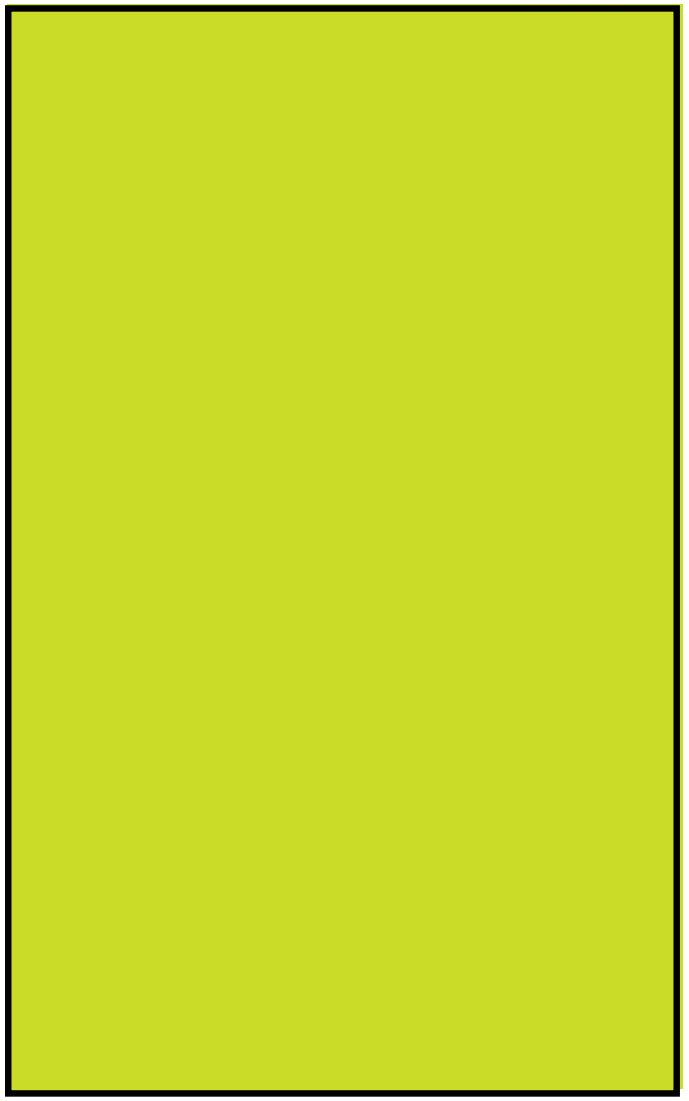
- Provide a number line with a known location and no endpoints.

- Identify possible endpoints using relationships between numbers.

The arrow is pointing at -13.
What are the endpoints?



What is a routine for number sense and reasoning?

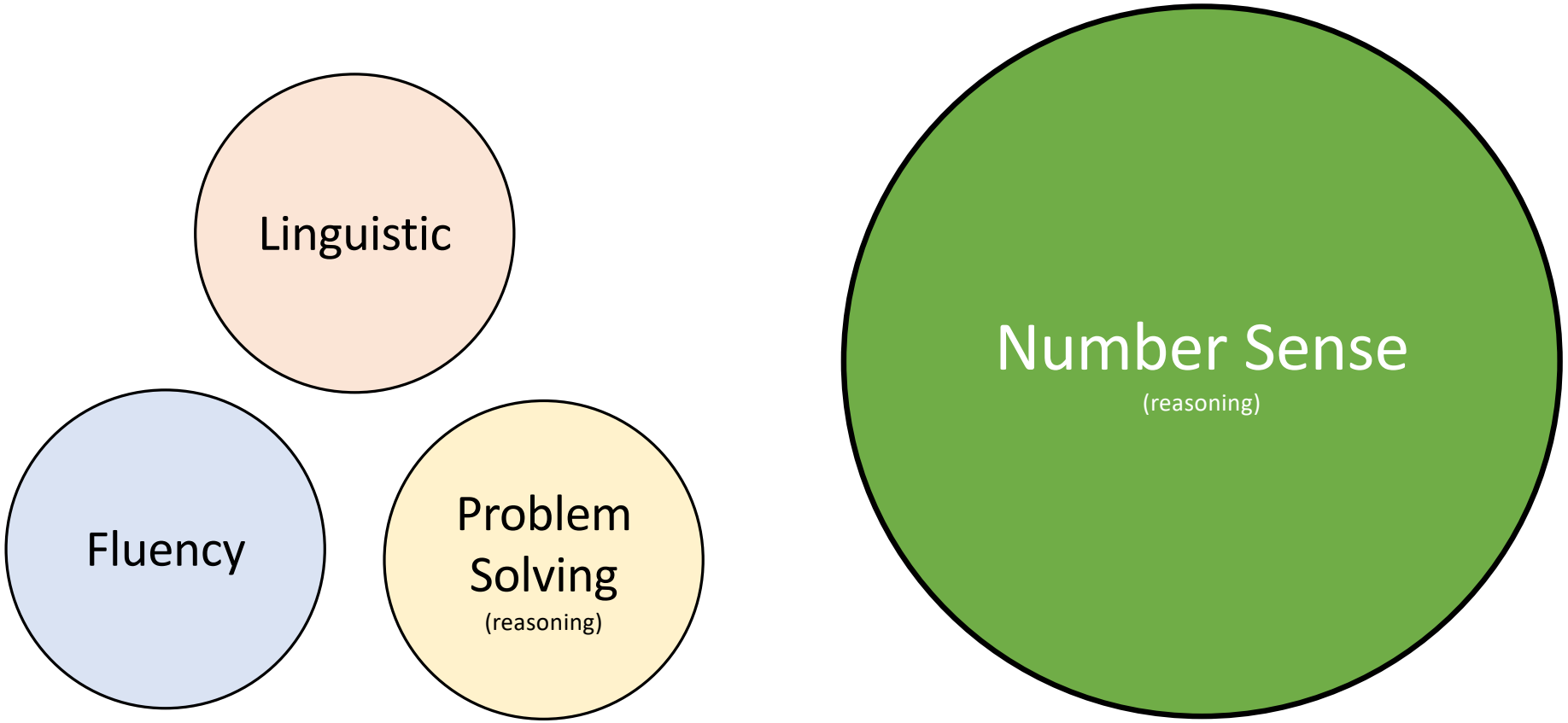


What is a Routine?

- A familiar, adaptable protocol for engaging students in learning through thinking and discussion.
- Establish expectations for engagement and participation
- Support management and foster positive mathematics relationships
- Are effective for reinforcing underemphasized skills.

Bay-Williams & SanGiovanni (2021)

Types of Instructional Routines



A Daily Number Sense (and reasoning) Routine?

- Brief (5-10) minute activity for promoting engagement, reasoning, and discourse
- Intends to reimagine the way we begin mathematics class (HW/WU)
- Develop number sense and reasoning

10,000 hours

10,000 hours	÷	180 days	= 55.6 years

If *just* this year….

$$\underset{\text{min}}{10} \times \underset{\text{year}}{1} \times \underset{\text{days}}{180} = \underset{\text{minutes}}{1{,}800}$$

$$\underset{\text{min}}{1{,}800} \div 60 = \underset{\text{hours}}{30}$$

Everyday

Every grade

If only minutes

K-8

$$\underset{\text{min}}{10} \times \underset{\text{years}}{9} \times \underset{\text{days}}{180} = \underset{\text{minutes}}{16{,}200}$$

$$\underset{\text{min}}{16{,}200} \div 60 = \underset{\text{hours}}{270}$$

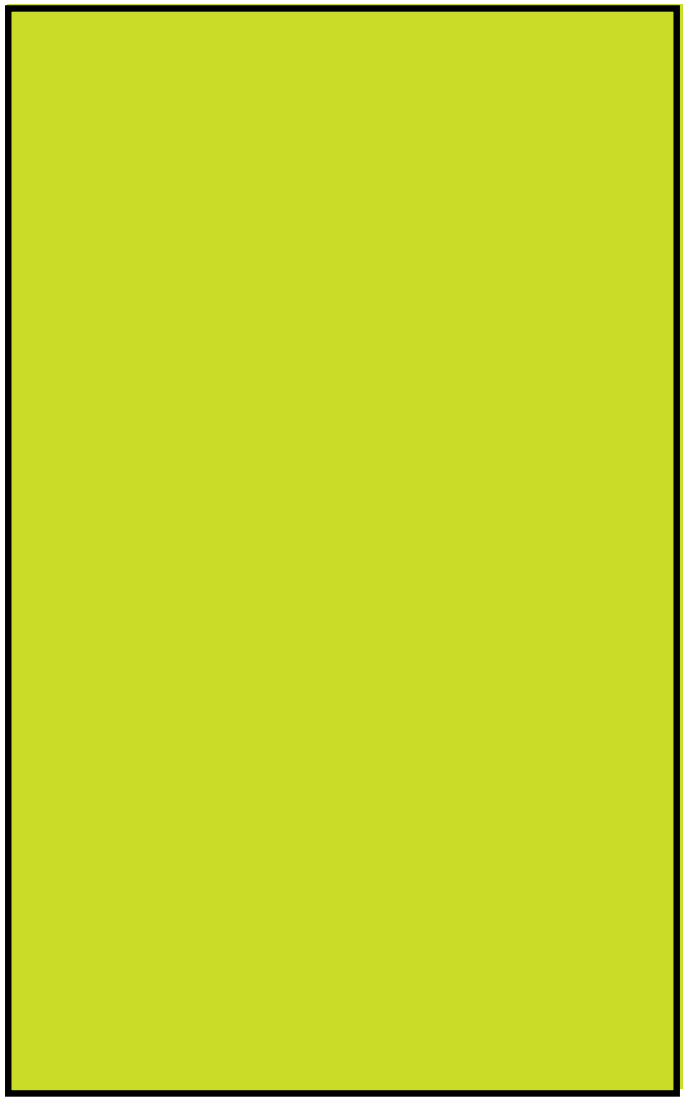
1 ½ school years

Everyday

Every grade

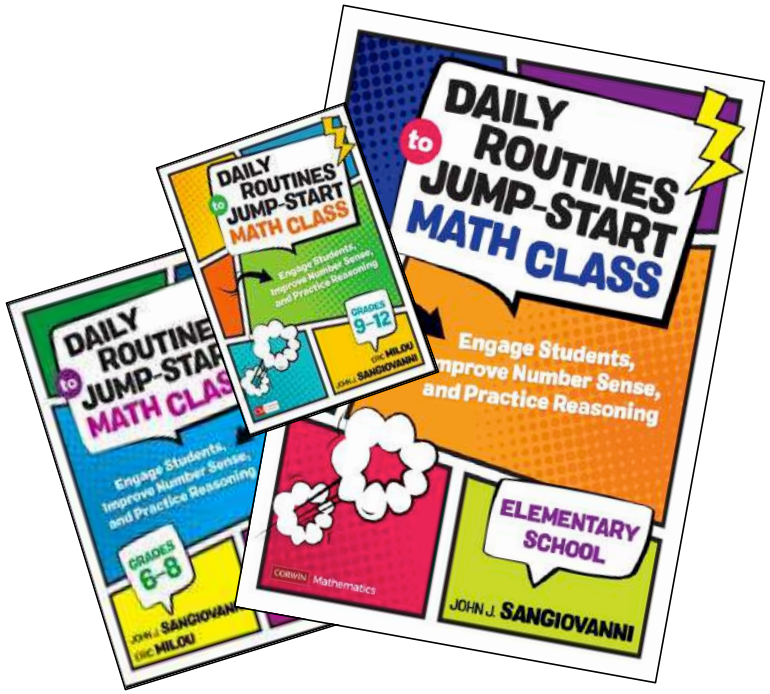
If only minutes

How do I facilitate a routine for number sense and reasoning?



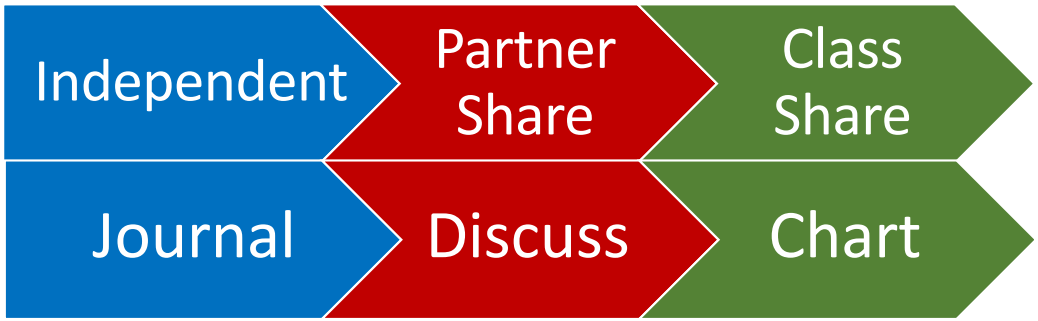
Routines: When?

- 5-10 minutes but no more than 8-10 minutes
- A math lesson could begin or conclude with a routine.
- In a block schedule, a routine might occur midway through the time period.



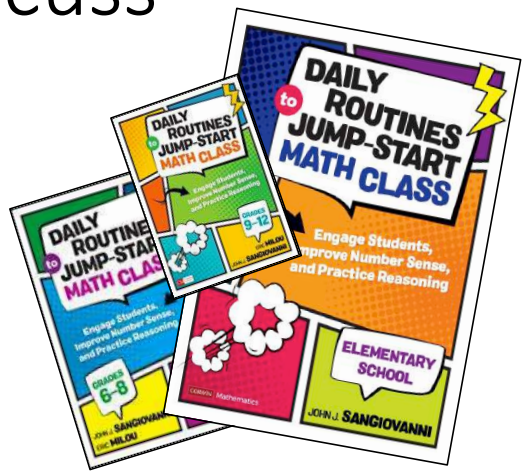
Routines: Facilitating a Routine.

- Students engage mentally
- Students share with partners
- Group share
- Teacher records and facilitates BUT does not influence or dictate



Routines: Timing

- 5 – 10 minutes
- About* 2 minutes for students to reason
- About* 1 minutes for partners to discuss
- About* 3-5 minutes for the class to discuss

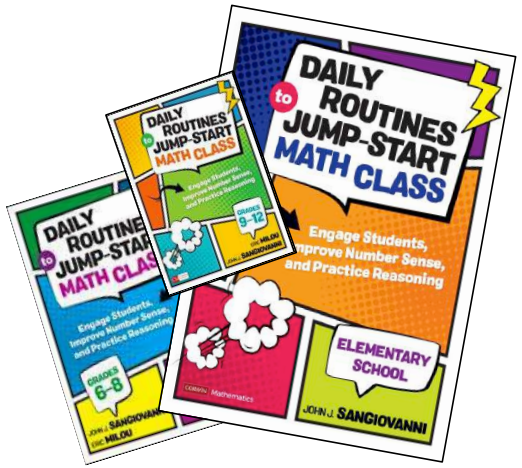


Routines: Keep in Mind

- EVERY student doesn't have to share
- EVERY "solution" doesn't have to be explored
- EVERY prompt doesn't have to be offered

Partners: Triads

Partners: Extended



This or That?

Deal A 	Deal B 
36 for $39.99	18 for $15.47

This or That?

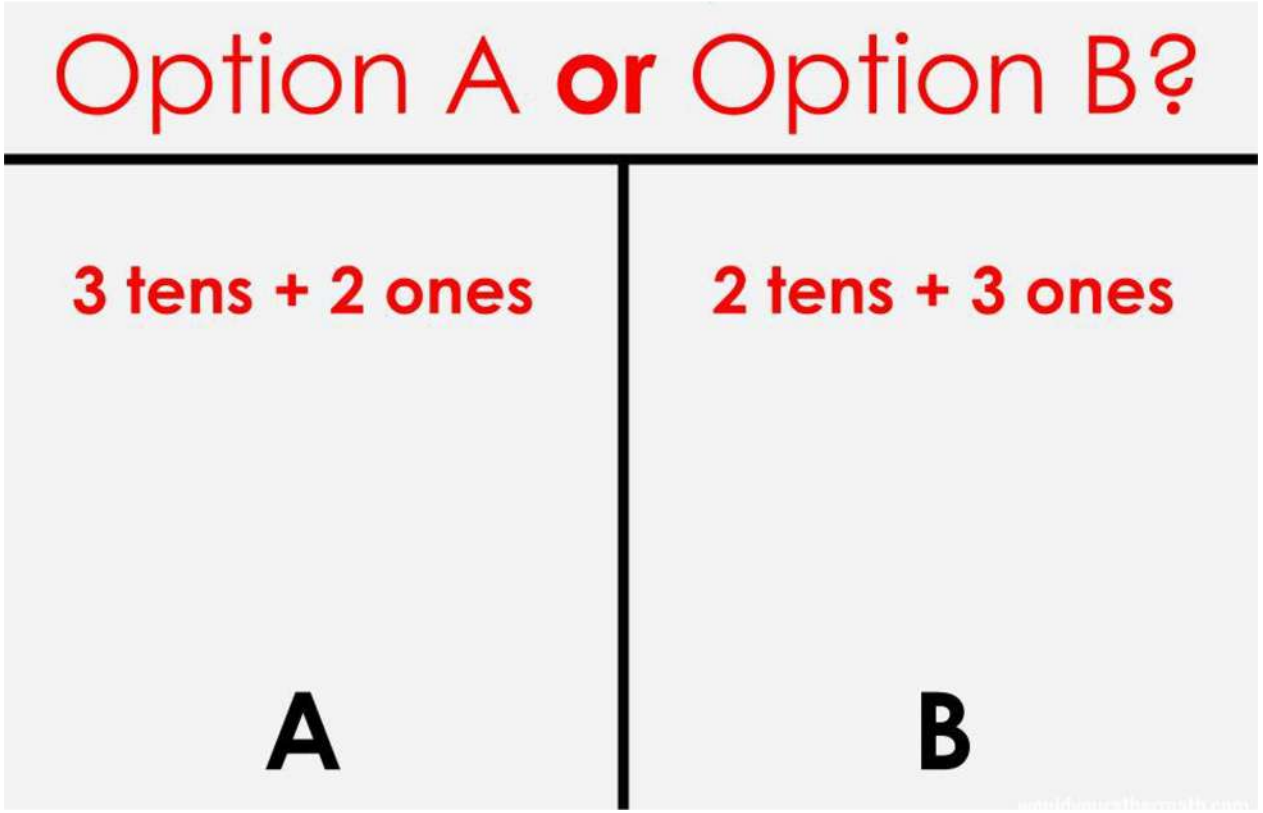
Deal A 	Deal B 
36 for $39.99	18 for $15.47
60 for $70.04	48 for $49.99

This or That?

4 boxes of these

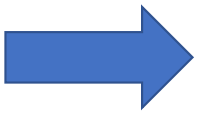
8 boxes of these

Would You Rather...



Ice Cream Sandwiches

THIS	THAT
3 x 6	20
4 x 6	9 x 4
5 x 8	6 x 9



Ice Cream Sandwiches

THIS	THAT
800 - 209	314 + 318
700 - 619	35 + 47 + 50
17 + 23 + 30	17 + 30 + 25

This or That?

Box A 	Box B 
39 x 48	2,000
419 x 8	2,000
12 x 20 x 10	2,000

Hours of Recess

THIS	THAT
700 ÷ 76	523 ÷ 25
814 ÷ 30	64 ÷ 2
517 ÷ 50	8 x 41

Two Columns

Is column A or column B greater? Is it too close to tell?

Column A	Column B
$8 + \frac{1}{2} - \frac{3}{4}$	$8 - \frac{5}{8} + \frac{3}{4}$
$63 \div 9.75$	$21 \div 2.99$
-49×4	$-101 \div -31$

Raffle Winnings

Column A	Column B
80% of $25	25% of $80
92% of $43	90% of $50
63% of $12	107% of $5

G. Two Columns

Is column A or column B greater? Is it too close to tell?

Column A	Column B
-530 - 200	-628 + 30
-8 + 73 - 12	-50 + -21– -91
-49 x 4	-101 ÷ -31

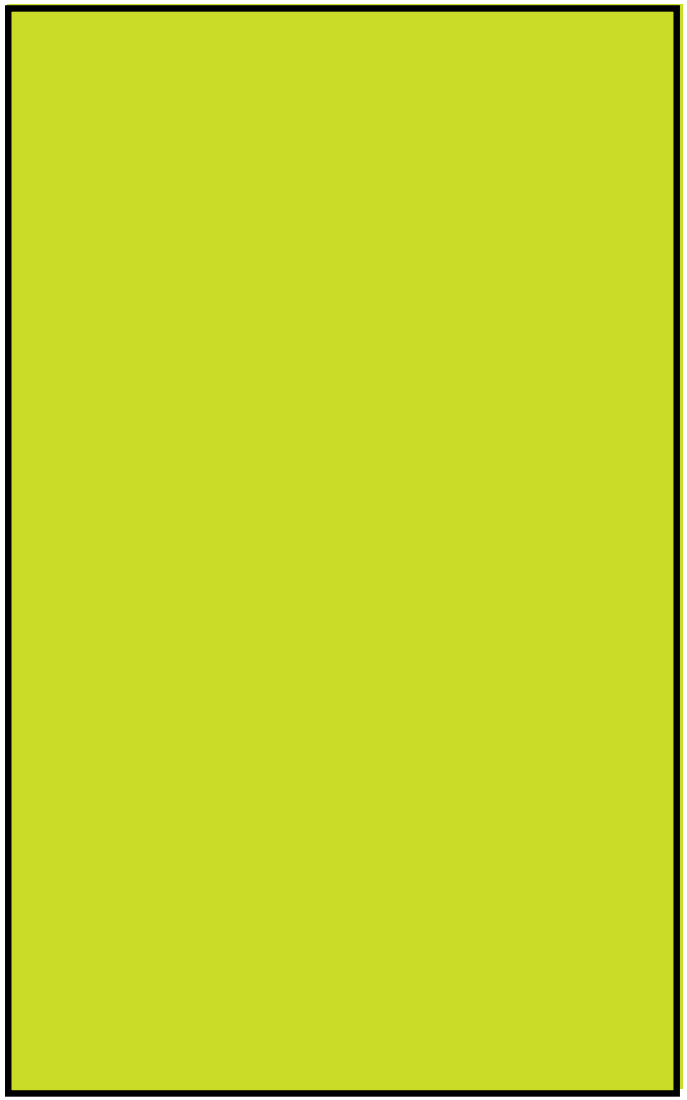
This or That (Two Columns)

- Pose two values to compare.
- Have students determine which they would select and why.

Ice Cream Sandwiches

THIS	THAT
3 x 6	20
4 x 6	9 x 4
5 x 8	6 x 9

What are one idea that resonate with you today?



Number Sense and Reasoning (4 - 8)

Minneapolis Public Schools
August, 2021

John SanGiovanni
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@JohnSanGiovanni

Where's the Point?

- What number is the arrow pointing to?
- How do you know?

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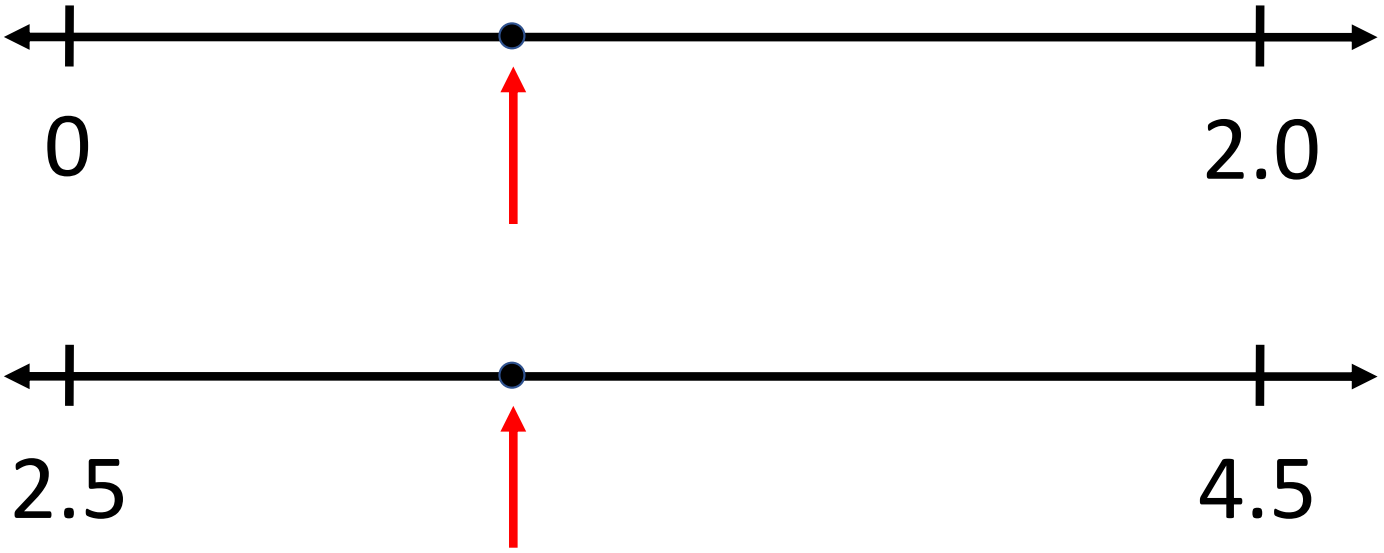

Where's the Point?

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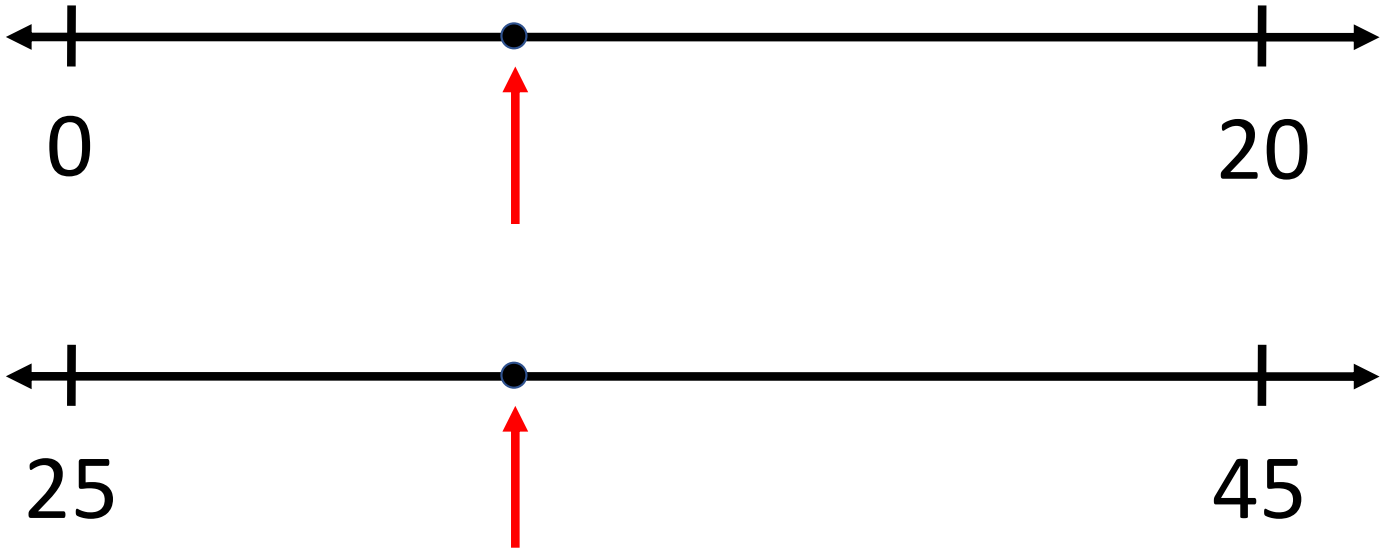
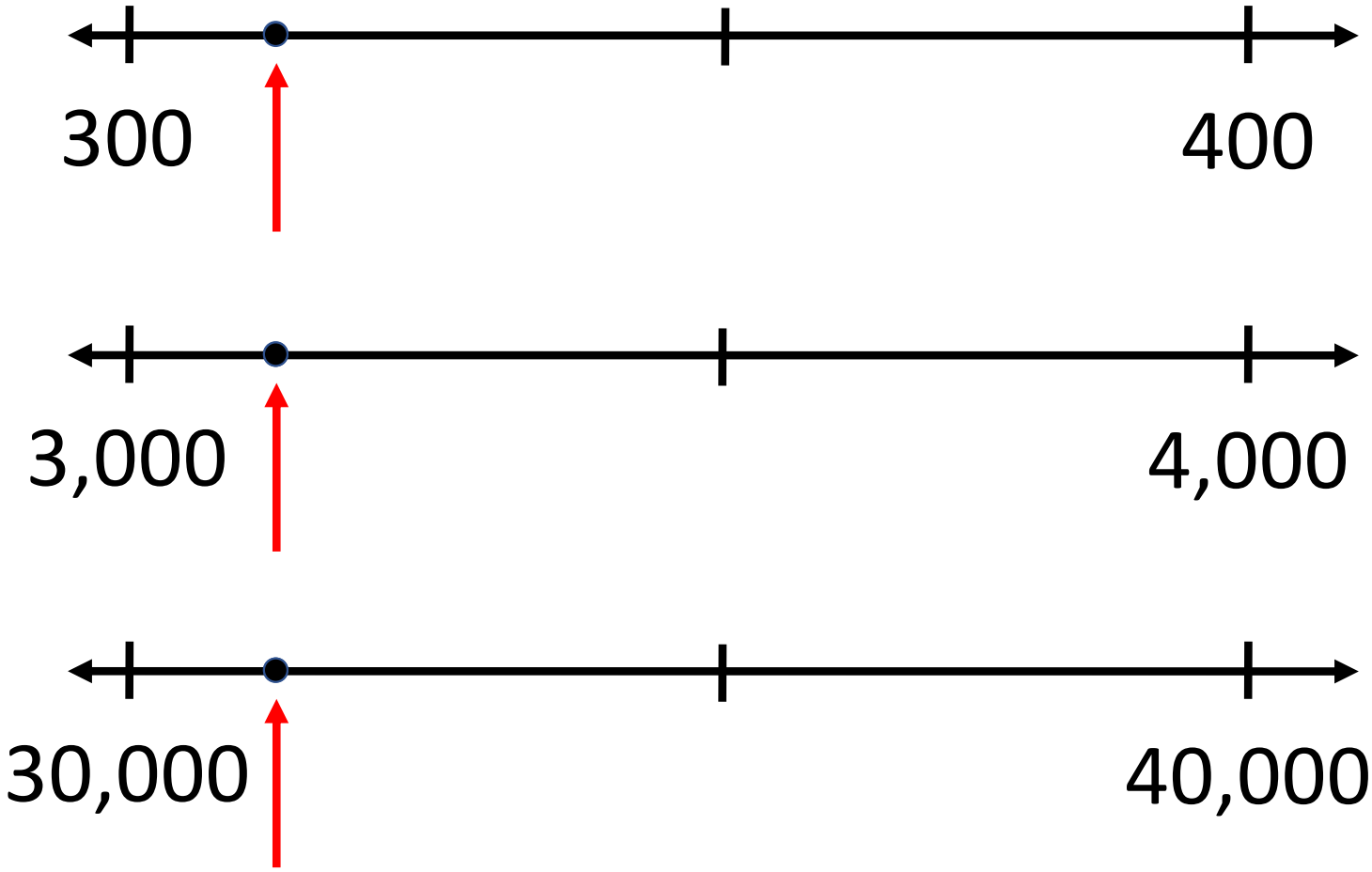
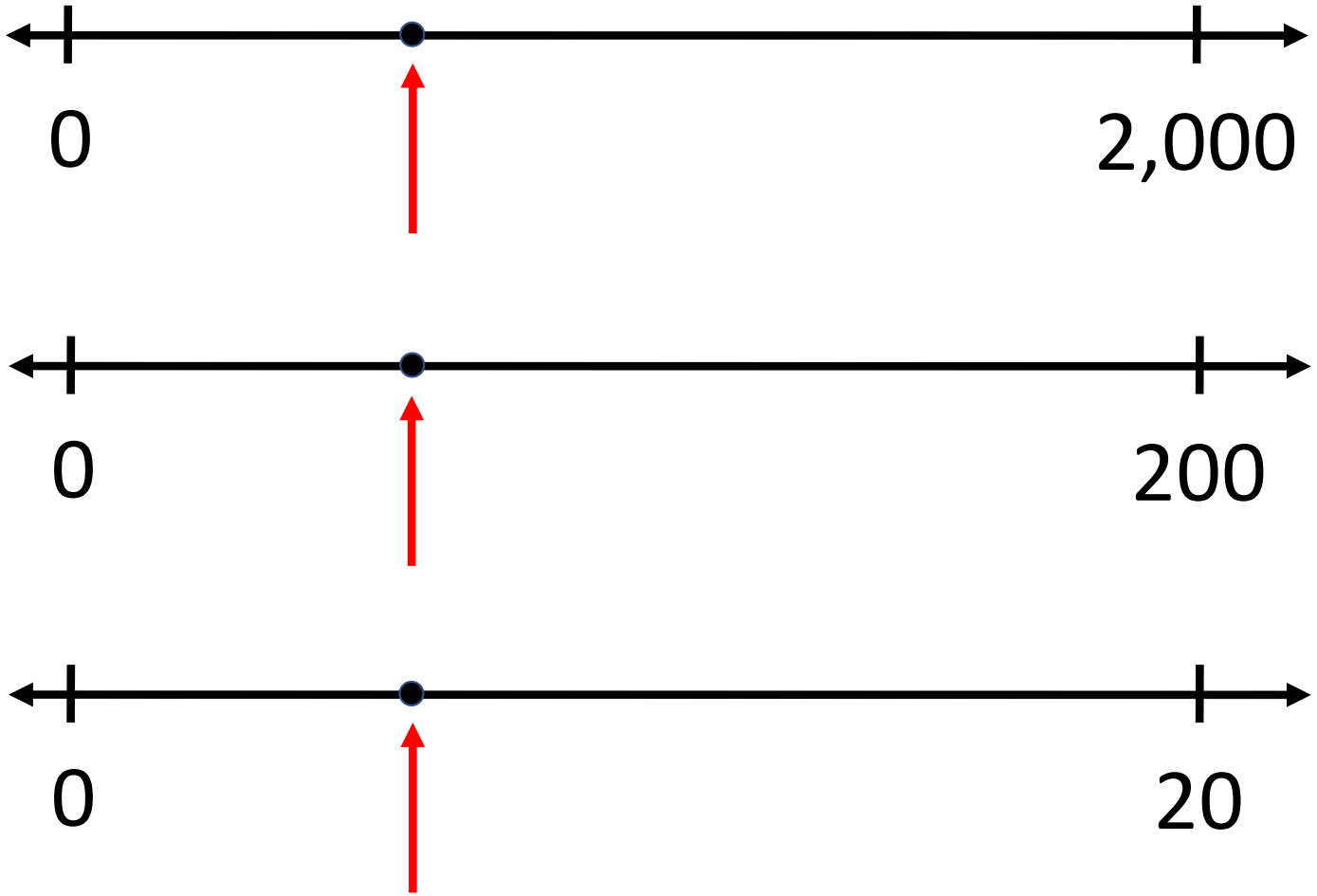


Where's the Point?

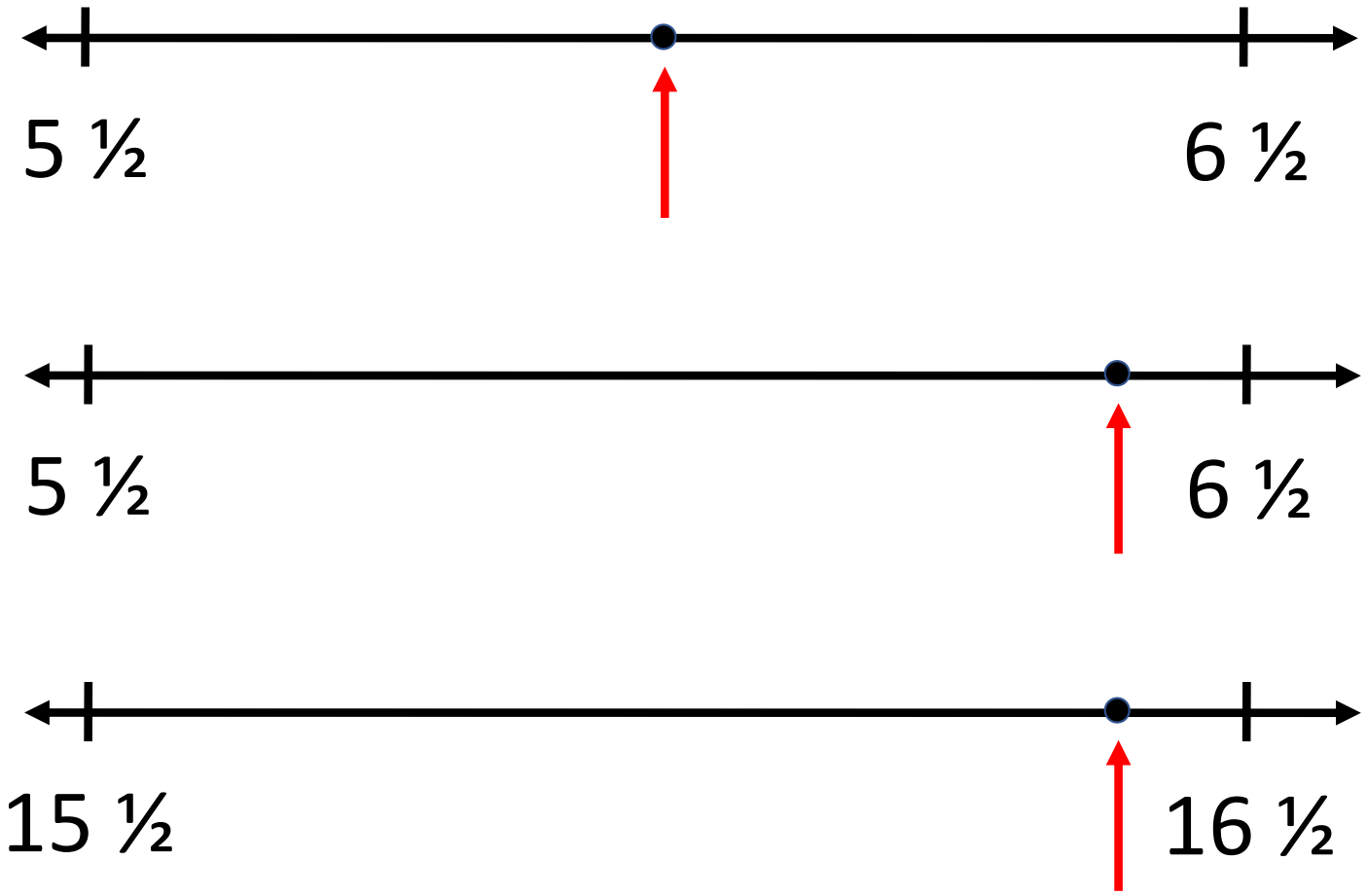



0
2,000
0
200
0
20

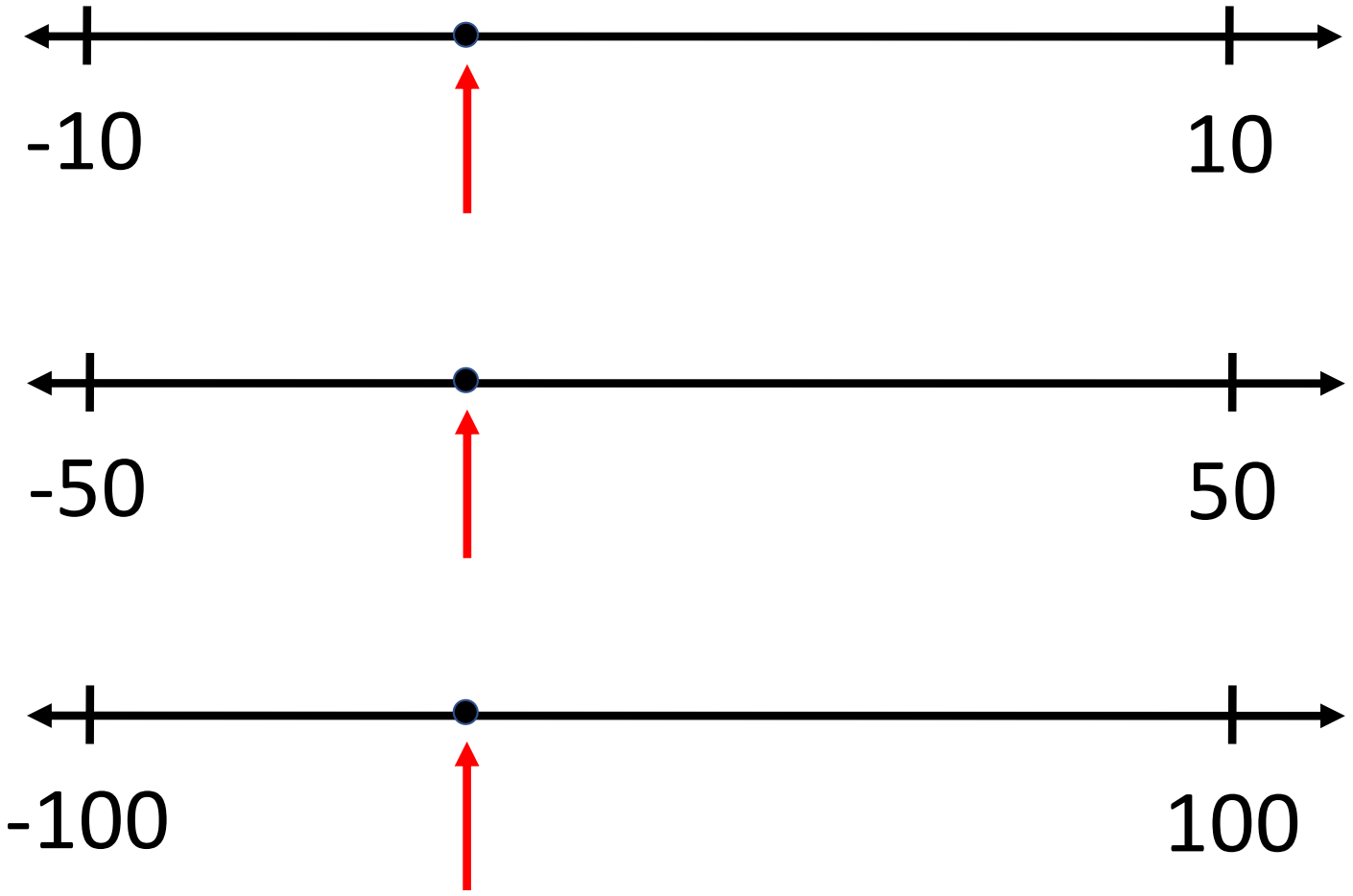

0
20
0
.2
0
.02

Fractions

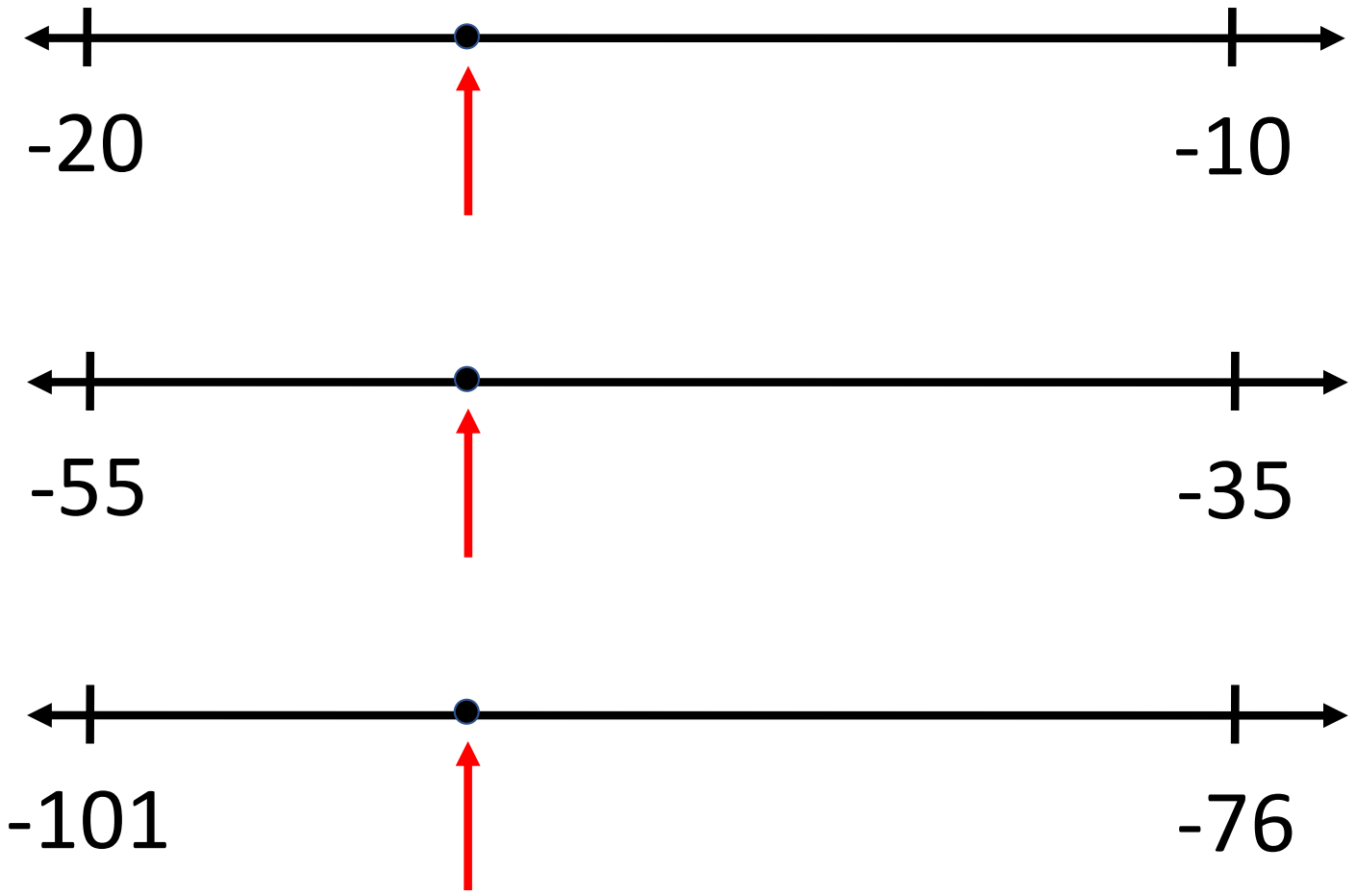
H. Where's the Point?



E. Where's the Point?



F. Where's the Point?



Where's the Point?

- Provide number lines with diverse endpoints.
- Identify the value of a specific location on the number line by reasoning about the relationships.